“Work in Progress:

do not quote,

 unless favorably.

Harrod’s Dynamics *vs.* Neoclassical Growth Theory

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1. Background.

The aim of this paper is to translate the macroeconomic model in my chapter in Bruce Kaufman’s book from a static context to a dynamic one. My model, embarrassingly, assumed that the “Keynesian short run” prevailed: that is, the stock of fixed capital is taken as given even though accumulation is occurring! Unfortunately, this problem cannot be solved by using the received growth theory of the neoclassical (NC) genre (e.g., Solow, 1956) without major modification. On top of its other important unrealistic assumptions, that theory assumes continuous full employment (FE). In contrast, my model requires that involuntary unemployment persist.[[1]](#endnote-1)

(Unfortunately, I did not finish the first part of the paper soon enough to significantly write the second part (part III, below). So it’s not included here.)

This means that Harrod’s (1939) Keynesian analysis is a better starting point. As Hoover (2008) argues, Harrod’s *Essay* *in* *Dynamic* *Theory* presents a very different theory from that of Solow, a difference that Solow himself misses. Hoover’s analysis fits well with the necessary rethinking of macrotheory after the Great Recession of 2007-09 and its stagnant aftermath.

This paper applies two tools to understand Harrod’s analysis (developed in Devine, 1980), i.e., the use of different “runs” (instead of using distributed lags or the assumption of movement to a posited long-term equilibrium) to analyze dynamics and the emphasis on the mutual interaction between the micro- and macro-levels and the need for consistency between the two levels (instead of a one-sided emphasis on “microfoundations”). One key contribution is that compared to both Harrod’s *Essay* and NC models, however, I drop the common assumption that growth at Harrod’s “natural” rate is the same as growth characterized by FE.

Harrod’s work requires examination not because it presents new ideas but because its old ideas have been shelved and forgotten. After he wrote his *Essay*, a major intellectual sea-change swept macroeconomic theory: his vision – and that of the Keynesian school in general – was almost completely replaced (at least among academic macroeconomists) by that of the Walrasian or NC school. Nowadays, even the so-called “new Keynesian” school tries to force Keynesian economics into the Procrustean Bed of Walrasian general equilibrium. The main difference from the “new Classical” school is that they add one or two “imperfections” to explain the absence of market clearing.[[2]](#endnote-2) Most often this involves the nominal stickiness of wages (cf. Davidson, 1992). The general success of this intellectual counterrevolution helped to convince most economists that Solow’s (1956) growth model had totally superseded Harrod’s dynamics. Thus, they view the *Essay* through a purely NC lens – or do not read it at all – and so miss its major points (cf. Hoover, 2008). Because the *Essay* was almost completely Keynesian and the current orthodoxy in macroeconomic theory is NC in the extreme (cf. stochastic general equilibrium theory), it is useful to start with an explanation of the Keynesian framework.

* 1. Keynesian Macroeconomics.

As Hoover (2008) notes, while Solow focuses on “growth,” i.e., ideal steady-state growth of a utopian economy that always enjoys FE, Harrod’s *Essay* concerns “dynamics,” i.e., processes occurring in moving equilibrium (“warranted growth”) which may or may not be at FE. While Harrod’s theory might include Solow’s as a subset, having perpetual FE excludes most of the former’s analysis from the start. Further, while Harrod’s analysis had some normative content (e.g., in defining the “proper” or desired rate of growth),[[3]](#endnote-3) his main aim was to develop a positive theory of dynamic macroeconomic supply/demand relations. This contrasts with the highly unrealistic assumptions that make NC growth models almost purely normative (considering such issues as “Golden Age” growth), strongly leaning away from positive and empirical topics.[[4]](#endnote-4) This explains why so many early articles in this genre referred to models as “parables” and the like; their authors had no pretenses about describing reality.

Unlike the NC tradition, remember that Keynesian models concern fluctuations in the general degree of factor utilization, from extreme underemployment of both labor-power and fixed capital goods to their over-use above FE.[[5]](#endnote-5) Keynes’ theory is called a “general” one because its application is not restricted to the special case where both the aggregate real net domestic product demanded (**D**) and the amount actually produced (**Y**) equal the level of output at FE (**F**). Further, having encountered such conditions during the Great Depression (which was ending as Harrod wrote), the original Keynesians often assumed that interest rates were mostly unresponsive to monetary conditions and/or that spending was usually insensitive to interest-rate changes.[[6]](#endnote-6) Since the “Keynes effect” (with falling prices causing **D** and **Y** to increase) could only be expressed via interest-rate changes, Keynesians rejected its empirical relevance. Third, perhaps remembering the disastrous results of deflation at the onset of the Depression (cf. Fisher, 1933; Tobin, 1975), they also rejected real balance (Pigou) effects. Thus, to Keynesians, the main force for macroeconomic change, including equilibration, is quantity adjustment (and not price adjustment), except when the economy exceeds the “inflation barrier” (with **Y** > **F**) in which case inflation soars.

When Solow (1956: 66) remarked that both the *Essay* and a related article by Domar (1946) consistently studied “long-run problems with the usual [i.e., Keynesian] short-run tools,” he revealed another profound division between the two visions, i.e., the meaning of the term “long run.” To the NC school, the long-run situation is a unique and exogenously-given equilibrium, with what’s been called the “natural” level of output (a.k.a. potential) which corresponds to the Friedman’s “natural” rate of unemployment (more scientifically known as the NAIRU). This equilibrium is assumed to be totally independent of both the short-run situations encountered on the way there (as with Wicksell’s “natural” rate of interest). The character of the long-run state is determined by exogenously-given values of supply-side variables, i.e., factor supplies, technology, management skills, and perhaps some factor market frictions and inefficiencies.[[7]](#endnote-7) The presumed independence of the long-term equilibrium growth path from short-run dynamics allows Solow and his intellectual descendants to analyze the former while ignoring the latter.

Second, in this view, the system gravitates to this equilibrium if markets are perfect and they are allowed to run free. In sum, the NC school embraces the axiom that with unfettered competitive markets, “in the long run, we’re at full employment” – or, more accurately, “we’re at the ‘natural’ level of employment.” Thus, in NC macroeconomics, the main controversy concerns how much time this adjustment takes and how complete it is.[[8]](#endnote-8) Then, the key policy issues are about the essentially microeconomic questions of how to raise “natural” employment and output by eliminating or moderating the effects of frictions and inefficiencies. To the current policy consensus, this mostly involves increasing “flexibility,” i.e., the abolition of non-market institutions, especially in labor-power markets.

Keynesians, in contrast, see the character of long-term results as determined endogenously by a series of short periods under conditions of fundamental (Knightian) uncertainty, with the present but a point in the relentless transition from the irreversible past to a unknowable future (Kalecki, 1968: 263; Robinson, 1980). Not only do hysteresis (path dependence) effects play a role in changing the nature of the long-run state (including as the values of both the NAIRU and potential), but Keynesian macroeconomics does not assume *a* *priori* that the level of demand will ever achieve the long-term equilibrium state merely via market adjustment. As noted, the original Keynesians saw the automatic price adjustment that is supposed to promote **D** = **Y** = **F** as very weak if not nonexistent (if unemployment is high). Thus, a persistent less-than-full employment situation, such as seen in the U.S. during the stagnant aftermath of the Great Recession of 2007-09 (with actual GDP growing steadily with, but significantly below, potential), might fit Keynes’ aphorism that “in the long run, we’re all dead.” Even a new Keynesian such as Lawrence Summers has suggested that the U.S. may be suffering from “secular stagnation.”[[9]](#endnote-9)

In sum, while the NC school emphasizes supply-side inefficiencies that exist when **D** = **Y** = **F**, Keynesians stress the role of inefficiency in the degree of resources utilization due to aggregate demand failure (**D** = **Y** < **F**). Because they do not dismiss the role of either physical limits on production or supply-side inefficiencies, they see the macroeconomic behavior as *asymmetrical*: unlike in Friedman’s (1968) NC theory of employment, for example, the result of operating above the inflation barrier is not a simple mirror image of what happens when low aggregate demand prevails. One aspect of this asymmetry is the *price/wage* *ratchet* *effect*: prices and money wages – and inflation rates – rise relatively easily but are “sticky” downward. Though the ratchet seems to have been broken during the 1920s, allowing destructive deflation during the early 1930s, this effect might prevent or delay future disasters.

* 1. Harrod.

Harrod’s understanding of above-full employment situations is consistent with NC analysis: as with other Keynesians of the era, he was fully aware of the inflationary consequences of going beyond the barrier set by **F**. In fact, as if aping the NC school, he refers to the growth rate of output at FE (**gf** = ∆**F**/**F**) as “natural.” However, Harrod followed the Keynesian tradition to examine the medium run before turning to longer-run analysis.

Harrod’s *Essay* assumes the simplest Keynesian Cross (KC) model. On the demand side, planned net fixed investment (**I** = ∆**K**) is the only “injection” into the circular flow of income, spending, and output, i.e., the only element of spending independent of the current value of **Y**.[[10]](#endnote-10) On the other hand, (net) saving (**S**) = ***s***·**Y** is the only “leakage” from that flow, while saving has no autonomous component. **S** may include the government’s budget surplus and the country’s trade deficit (i.e., net financial inflow) in addition to private domestic saving, just as **I** may include government investment. However, I do not bring these in here.

I see Harrod’s main contribution as being on the supply side. He aimed to extend the scope of the application of KC models. The latter had been restricted to the Keynesian short run (KSR). In that “run,” even though investment helps to determine **Y**, it has no impact on the useable stock of capital goods (**K**) or output at full capacity utilization (**Z**). He replaced KSR analysis with that of (what I call) the *Harrodian* *Medium* *Run* (HMR): during this run, both **K** and **Z** typically rise. As in non-NC accelerator models of investment, those two variables are linked by a fixed capital coefficient or capital-output ratio (***c***). During the HMR, ***c*** is constant and equal to the incremental capital-output ratio. (The HMR seems to be on the order of a calendar year.) This in turn means that the increase of capacity is determined by net fixed investment (**I**):

|  |  |
| --- | --- |
| ∆**Z** = ∆**K**/***c*** = **I**/***c*** | (1) |

In the HMR, Harrod also assumed that the saving rate (***s***) was constant. Below, I show how ***c*** and ***s*** determine Harrod’s warranted rate of growth of **Y**:

|  |  |
| --- | --- |
| **gw** = ***s***/***c***  | (2) |

In §II.A.1, I derive this equation starting from a simple flexible accelerator model and then adding four ideal assumptions. One key result is that for Harrod’s moving equilibrium with **g** = **gw** to make sense, we must also assume full capacity utilization.

Though this growth rate represents a moving equilibrium, it is a “knife edge” (unstable): if the economy starts on this path and there are no exogenous shocks, the economy will continue on the path, but any exogenous shocks cause either an explosion or an implosion as deviations of the actual growth rate (**g**) from **gw** cause positive feed-back. Limits on these disasters arise only from factors outside of the model behind equation (2).

In his *Essay*, the value of Harrod’s two key coefficients can and do change over time. The ***c*** even changes “as income grows and in different phases of the trade cycle.” In fact, just as in Solow, it “may be somewhat dependent on the rate of interest.” I interpret such changes as occurring *between* HMR periods, i.e., during the *Harrodian Longer Run* (HLR).[[11]](#endnote-11) In these terms, Harrod’s analysis is very relevant to moderate followers of the NC model who believe in the factor substitution that is so necessary to that model’s equilibration but do not see such adjustment as happening instantaneously. The distinction between the HMR and the HLR can be used to capture the relative “stickiness” of factor proportions. Of course, people may differ about how much calendar time corresponds to these “runs.”

* 1. Limits.

One problem with Harrod’s discussion is his over-stress on the role of saving. This made it easy for some to conclude that he saw **S** as causing **I**, just as in NC models. Following the Keynesian tradition, I emphasize the independent role of the latter (cf. Sen, 1970). As Keynes’ (1936: ch.16) argued, **S** does not automatically drive **I**. In a non-barter economy, saving is nothing but the accumulation of new paper promises (securities and money).[[12]](#endnote-12) Then, the real volume of saving *limits* the flow of funds allowing investment to occur if and only if the level of **Y** is given.[[13]](#endnote-13)

Next, I emphasize the role of the capacity utilization rate (***y*** = **Y**/**Z**), which Harrod almost always left backstage (unlike Domar, 1946). Unlike in much of the literature, full-capacity output (where ***y*** = 1) should not be seen as an absolute limit on output. Instead, it corresponds to the output level where average total production costs are minimized (absent demand constraints on production). In an economy producing only services and manufactured goods (i.e., no primary products), as ***y*** rises up to unity during the KSR, both the average variable and marginal cost curves are horizontal. Then, for ***y*** ≥ 1, both rise steeply.[[14]](#endnote-14) This flatness occurs due to the fact that the employment of labor-power rises in step with the utilization rate of fixed capital: diminishing returns to labor’s use of **K** do not occur until ***y*** ≥ 1 (Kalecki, 1943). Thus, inflation becomes increasingly more likely to replace real increases. This result is reinforced by price behavior in primary-product sectors (which suffer from diminishing marginal returns due to the limits imposed by Nature). However, inflation is moderated to the extent that businesses maintain unused capacity for flexibility.

Further, though ***y*** = 1 has the lowest average total costs, this rate is not always achieved: **D** imposes a quantity constraint on **Y**. For given **Z**, this determines ***y*** in KC equilibrium. Demand also constrains labor-power markets, determining the employment ratio (***e***).

Finally, to Harrod, ***c*** is a ratio between two nominal magnitudes and thus varies with the relative price of capital goods. But that possibility is ignored here. I follow the long NC tradition (which seems to have been initiated by Solow) of totally abstracting from the real-world heterogeneity of capital and other goods.[[15]](#endnote-15) That is, in the “corn-corn” model presented below, labor uses a stock of inputs (**K**) made out of homogeneous “corn” to produce yet more “corn” which is then consumed or invested. In NC terms, production is done by a “representative firm” (RF), i.e., a standard tool used in developing “microfoundations” that totally ignores the aggregation problems. However, the demand-side aggregation problem called the “paradox of thrift” is not ignored below.

* 1. The FE Assumption.

A key problem is that the growth literature from Harrod to Hoover (with Solow in between) employed *the FE assumption*, i.e., that if **gw** = **gf**, the growth path is characterized by ***e*** = 1. That is, the growth literature assumes that the economy follows the FE path labelled ***FF*** in figure 1. But there is another possibility: this is a warranted growth path such as that marked ***YY***, with ***e*** constant but different from than unity.[[16]](#endnote-16)

This case fits Okun’s empirical generalization (or “law”), which says that **Y** can enjoy positive growth without lowering unemployment and raising ***e*** – or *vice-versa*. Because of the normal steady growth of the labor force and the way in which productivity growth tends to make current employees redundant, output growth must be positive merely to keep ***e*** from falling. In dynamic version of this “law,” if actual **Y** growth equals a given rate, then ***e*** is constant, but not necessarily equal to unity (as with movement along line **Y**). That given rate is basically an empirical estimate of Harrod’s **gf**. Then, if **g** < **gf**, ***e*** falls even if **g** > 0, as during a “growth recession,” an attempted “soft landing,” or a “jobless recovery” (as with the arrow starting at point **3**). Rising ***e*** requires **g** > **gf** as during a “true recovery” (as with the arrows starting at points **4** and **1**).

Figure 1: Two Possible Warranted Growth Paths.



Given the dropping of the FE assumption, a key question is the original Keynesian one, i.e., the determination of ***e***. To replace the KSR analysis of Devine (2015), I follow Solow to describe cases with **gw** = **gf** (as in Figure 1) with constant ***e***. On this level, the model is basically NC in nature. In fact, for argument’s sake, production is portrayed in an ultraorthodox way, i.e., by an aggregate Cobb-Douglas production function with constant returns to scale.

Unlike Solow, however, I do not assume that the actual growth rate is determined by the warranted rate. Instead, causation between **g** and **gw** goes both ways. First, **g** is explicitly driven by investment which is independent of saving. Using an investment function more complicated than the simple accelerator relations that Harrod assumed and I present in the appendix, investment is determined by expected profitability. Thus, investment and **g** vary with ***y*** (among other things). Going the other way, ***e*** and ***y*** are assumed to move together and are directly determined by demand growth (the actual **g**). The two-way interaction between investment and **g** on the one hand, and the two utilization rates on the other implies equilibrium, determining both ***y*** and **g**.

1. Harrod *vs.* Solow, an Interpretation.

As Hicks (1949) notes, “no one can study Mr. Harrod’s work at all deeply without feeling that results of really great significance are just round the corner.” Like him, I follow that feeling. However, Hicks erred when equating Harrod’s work with accelerator/multiplier models of the type that Samuelson (1939) made famous. The problem is that this approach abolishes or ignores the distinction between warranted and actual growth.

Unlike Hicks, I aim to ferret out the exact meanings of the key terms, i.e., “warranted” and “knife edge” in Harrod. Like the latter, I abstract from lags in the accelerator/multiplier relationship (even though they clearly are empirically important). In my view (as for Harrod), lags are part of *un*warranted growth and thus mostly outside of the analysis.

As Hoover (2008) points out, Harrod’s main question differed drastically from that of Solow (see also Sen, 1970). This concerned the unstable nature of a moving equilibrium where **g** = **gw** while for Solow, the focus is on the relationship between an assumed-to-be stable growth path with **g** = **gw** on the one hand and **gf** on the other. That is, while Solow described what happens when the economy stays at FE, most of Harrod’s analysis concerns a world in which **g** usually does not exceed the speed limit set by an exogenously-given **gf** for a period long enough to attain or cross the inflation barrier at **F**.

Of course, as discussed in §II.B, Harrod did consider those cases where growth occurs at or above full employment. As discussed there, growth above **F** automatically implies forces that reduce the growth rate while (in the Keynesian spirit of asymmetry) growth below **F** does not imply automatic movement up to full employment. In the present section, however, the focus is only on the case of growth where output **Y** is less than **F**.

* 1. Harrod’s Knife Edge in the HMR.

The Keynesian framework starts with the independence of investment from saving. To describe this, I use a moderately realistic investment function from the flexible accelerator family[[17]](#endnote-17) to explain the exact meaning of warranted dynamics in which everything “goes right.” Harrod himself clearly had an accelerator-type model of investment decisions in mind. Though he saw lags as implying deviations from the ideal moving equilibrium, the fact that they are so familiar to most modern macroeconomists makes them useful as a starting point. To aid clarity, assume that HMR conditions apply during a “year.”

* + 1. Warranted Growth.

Because ***c*** is given in the HMR, the equations below omit the role of changing investment due to variations in expected profitability, interest rates, and the like. This is intentional: the point here is not to describe actual behavior but instead to limn the conditions defining Harrod’s moving equilibrium.

**(a)** During a year, the RF’s desired level of investment (**ID**) is the amount needed to fill the *capital* *gap* between the desired capital stock (**KD**) at the end of the year (at time **t**) and **K–1**, i.e., the stock existing one year earlier (**t–1**). The former is determined by both the capital-output ratio and the expected level of output ($\hat{Y}$) at time **t**.[[18]](#endnote-18)

|  |  |
| --- | --- |
| **ID** = **KD** – **K–1** = ***c***·$\hat{Y}$ – **K–1** | (3) |

The capital gap can be positive because output is expected to grow relative to **Y–1** and/or because slow delivery and installation of capital goods have implied a low **K–1** compared to **KD** at **t–1**.[[19]](#endnote-19) A positive gap then implies positive desired net investment (more than replacing **K** lost to depreciation). In contrast, falling or low expected demand and/or capital equipment coming on line too late or surviving depreciation “too long” can cause capital overhang, i.e., a negative capital gap (**KD** < **K–1**). This may have been seen on the macro level in the U.S. during 2001 (Economic Report of the President, 2002, pp. 39-40).**[[20]](#endnote-20)** This type of gap implies negative desired net investment: the RF refrains from doing some replacement investment required to cover physical depreciation costs.[[21]](#endnote-21)

In a dynamic world, of course, firms rarely realize their desires: *actual* planned investment spending (**I**) depends on supply-side bottlenecks in the production, provision, and installation of new capital goods or the costs of cancelling their delivery and installation once the process has begun. In addition, Kalecki’s (1937) principle of increasing risk discourages the RF from plunging forward to fill the capital gap completely. In sum, **I** is described as partly determined by real or perceived adjustment costs, which move inversely with the adjustment coefficient (***θ***), which is assumed to be constant in the HMR. Thus, **I** = ***θ***·**ID** and:

|  |  |
| --- | --- |
| **I** = ***θ***·[***c***·$\hat{Y}$ – **K–1**] \_ with 1 ≥ ***θ*** > 0  | (4) |

**(b)** Next, to define what’s meant by everything “going right” (the warranted growth path) in this flexible-accelerator world, add four unrealistic assumptions to equation (4). These are akin to those normally made in models of the new Classical type. The first two assumptions describe a situation in which the RF is doing as well as it can, so that everything is “right” at the micro level. Decision-making is “optimal” and of course much better than in the real world of uncertainty and frictions. Then, the decision of the RF (with no ability to coordinate decisions with other firms) is “scaled up” to represent the investment decision of all firms on the macroeconomic level: if there are *N* identical firms, the total investment at a given time would be *N* times the investment made by the RF.[[22]](#endnote-22)

*First*, let ***θ*** = 1 so that capital gaps are closed over the year: **K–1** = ***c***·$\hat{Y}$**–1** and **I** = ***c***·Δ$\hat{Y}$. *Second*, assume that expectations are always *exactly correct*, with no random error. With such ultra-“rational” expectations (RE), the RF invests based on $\hat{Y}$ = **Y** and $\hat{Z}$ = **Z**. In line with the notion of everything going right, objective conditions (**Y** and **Z**) determine subjective predictions ($\hat{Y}$and$\hat{Z}$) rather than *vice-versa* (as can happen in Keynesian models).[[23]](#endnote-23) This gives the actual or investment-driven growth rate, which can be stated in two ways.

|  |  |
| --- | --- |
| **I** = ***c***·Δ**Y** **\_** and **\_** **g** = ∆**Y**/**Y** = (**I**/**Y**)/***c*** = ***α*y**/***c*** **\_** **\_**with ***α*y** = **I**/**Y** | (5) |
| Alternatively **g** = (**I**/**Z**)·(**Z**/**Y**)/***c*** = ***α***/***c***·***y*** **\_** **\_**with ***α*** = **I**/**Z** | (5’) |

These two assumptions complement each other because a RF with truly RE would be able to expect any adjustment costs or increasing risks and negate their role in practice.

Students of the accelerator model know that an investment function such as (5) is an endogenous source of instability since the lags that were assumed away moderate any investment surges or declines and also their (multiplied) effects on the economy. More than anything else, this fact seems to be the essence of Harrod’s knife-edge.

In the present case, the RF may “rationally” expect growth which is either excessively high or excessively low from a more objective perspective. That is, assuming RE does not imply the usual harmonious results in disequilibrium situations or when macro equilibrium is not unique and/or unstable (cf. Evans, 1983): acting on RE can encourage a non-optimal growth path to continue toward explosion or implosion.

But to define a moving equilibrium, we must rule out such instability by assuming that the macroeconomic context in which the RF operates also has everything right, with micro decisions and macro results being exactly in sync: the results of the scaling up the decision of the RF (determining the economy’s **g**) are totally consistent with macroeconomic conditions allowing this growth path to persist.

First, there is nothing in the story to assure that the right amount of demand exists to buy the product. **D** must equal **Y** at each point. For this to be true, the *first* macro assumption must be made, i.e., that KC equilibrium prevails with **I** = **S**. Thus, ***α*y**= ***s***. In turn, **g** also equals the warranted growth rate that Harrod derived (***s***/***c***) as seen in equation (2) above.

However, this assumption does not mean that ***y*** is constant. Growing at the warranted rate is only necessary, but not sufficient to having a stable growth rate. In fact, as far as I can tell, Harrod did not realized this, even though his analysis implies it. To see this, examine the rate of growth of ***y***, whichequals

|  |  |
| --- | --- |
| ∆***y***/***y*** = **g** – **gz** \_ where \_\_ **gz** = ∆**Z**/**Z** = (**I**/***c***)/**Z** = ***α***/***c*** | () |

By (5’), for ***y*** to be constant, **g** = ***α***/***c***·***y*** must equal **gw** = ***α***/***c***, so that ***y*** must equal 1. The *second* macro assumption is that this is true. This is like Say’s axiom, except that the assumed “full employment” applies only to fixed industrial capacity and not to labor-power.

Note that, unlike in NC growth models, saving does not determine investment while ***s*** does not determine ***α*y**. Instead, the exogenously-given ***s*** determines the accumulation rate which is *required* for KC equilibrium to occur with **g** = **gw** = ***s***/***c***. The equation does not imply that all actual growth paths that occur are warranted.

In sum, Harrod’s warranted growth rate describes a moving equilibrium of the accelerator/multiplier sort. Unlike in the accelerator-multiplier models of Samuelson (1939) or Hicks (1949), however, it is not the magnitudes of ***s*** and ***c*** that can imply instability: the moving equilibrium is unstable *even* *when* the values of ***c*** and ***s*** imply that a Samuelson-type accelerator/multiplier KSR model has smooth equilibration or damped oscillation. Instability arises instead due to the key element elided by KSR analyses, i.e., the growth of **Z** over time *due* *to* investment (cf. Kalecki, 1933). Turn now to the nature of this instability.

* + 1. The Knife Edge.

All four of the simplifying assumptions are likely to be broken in the real world so that **g** ≠ **gw** should be seen the norm. Here, drop only the ***y*** = 1 assumption while maintaining the others: the emphasis on cases where the actual growth rate equals the warranted one implies a stark contrast between micro rationality and macro results while still having KC equilibrium. Since KC equilibrium requires that ***α*y** equals the given value of ***s***, ignore the former symbol from here on.

**(a)** Our story centers on the *actual* and warranted growth rates, from above:

|  |  |
| --- | --- |
| **g** = ***α***/***c***·***y*** | (5’) |
| **gw** = ***s***/***c*** | (2) |

If **g** = **gw**, the actual economy has a growth rate equal to the warranted one, in what might be called the HMR equilibrium. Put a different way, for the actual growth rate to equal the warranted rate, the following condition must be met:

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| --- | --- |
| if \_ ***α*** = ***s***·***y*** \_ then \_ **g** = **gw** | (7) |

The latter formulation, unlike Harrod’s original one, says that the parameter ***c*** plays no mathematical role of in causing the two growth rates to differ.[[24]](#endnote-24) This is ironic: the key variable that Solow saw as allowing for equality of the warranted and “natural” growth rates (see §III.B.2) turns out to be unnecessary to the analysis of Harrod’s knife edge. This irony arises because Harrod and Solow were addressing significantly different questions.

Note also that condition (7) does not ensure equilibrium or that the growth will persist. As discussed above, having ***y*** = 1 is also necessary. Turn to this issue next.

**(b)** Just as important as the possible deviation between **g** and **gw** is the economy’s reaction to it. Does the economy move back into warranted growth (as in NC models) or is the equilibrium unstable (as for Harrod)?

As noted in § I.A, the answer depends on Keynesian quantity adjustment. This focuses on the implications of the investment function (equation (4)). To draw these out, first that the expected rate of capacity utilization ($\hat{y}$) equals $\hat{Y}$/**Z**. With **K–1** = ***c***·**Z–1**, we get:

|  |  |
| --- | --- |
| ***α*** = **I**/**Z** = ***θ***·***c***·[$\hat{y}$ – **Z–1**/**Z**]  | (8) |

For a given growth rate of full-capacity output (**gz**), **Z** = (1 + **gz**)·**Z–1**, so that:

|  |  |
| --- | --- |
| ***α*** = ***θ***·***c***·[$\hat{y}$ – {1/(1 + **gz**)}] | (9) |

The accumulation rate rises with $\hat{y}$ and/or the growth rate of capacity. Most likely, investment decisions depend on the *expected* growth rate of full-capacity output (part of what Keynes called the “state of long-term expectations”) instead.

Finally, assume that a generalized version of adaptive expectations applies for actual (non-ideal) investment behavior, so that:

|  |  |
| --- | --- |
| $\hat{y}$= $\hat{y}$**–1** + ***e***(***y*–1** – $\hat{y}$**–1**) + ***ε***\_ with ***e***(0) = 0 and ***e***’ > 0.  | (10) |

Here, ***ε*** is a random error term that represents other factors that can affect $\hat{y}$ such as Keynesian “animal spirits” (what might be called the “state of short-term expectations”) For the adaption of expectations (error-learning) to be the main tendency, it should have an average value of zero. In this context, no assumption about its statistical distribution is needed.

**(c)** Now use figure 2 to consider the two cases of warranted growth with ***y*** ≠ 1. The horizontal ***SR*** line represents the fact that ***s***is given in the HMR. The upward-sloping ***WG*** line represents all of the possible cases where condition (7) is met for a given value of ***y***, so that the actual growth rate equals the warranted rate whether or not capacity utilization is full. Finally, point **W** represents the specific case of warranted growth with ***y*** = 1 so that ***α*** = ***s***.

The cases with ***y*** ≠ 1 can be understood via a two-step procedure. In step (a), ***y*** is taken as given and constant. In step (b), causation goes in the opposite direction: deviations of ***y*** from 1 imply a changing ***y*** and ***α***, following equations (10) and (9).

Start at point **W** and step (a), with the small wobbles away from this equilibrium being exaggerated in the graph. Suppose that, due to a fall of ***α***, ***y*** falls to ***y*A** < 1, so that ***α*** = ***s***·***y*a** < ***s*** at point **A**. With ***y*** staying constantly equal to ***y*A**, we see growth at a constant rate despite the low value of ***y***. As with all points on the ***WG*** line, actual growth is warranted. That is, we see steady growth of **Y** exactly in step with **Z** but with the former always being lower than the latter, a relationship is similar to that between the growth paths ***YY*** and ***FF*** in figure 1.

Figure 2: Warranted *vs.* Actual Growth.



That this is merely a *temporary* equilibrium can be seen via a down-to-earth example: the RF adds a second factory (increasing capacity) while continuing to use both factories during only two out of three possible shifts. In this situation, total capacity grows but stays underused at a constant rate. In this case, unused capacity means that the accumulation rate is likely to fall. If the firm is standing in for the entire economy, falling accumulation reduces aggregate demand and thus ***y***. So we must move on to step (b).

This situation causes $\hat{y}$ to fall (following equation (10)) so that ***α***also falls (following (9)). Falling ***α*** also depresses **gz** and likely the value of that capacity growth rate expected to prevail. These depress ***α*** and in turn causes ***y*** to fall, but then it is possible that a new **g** = **gw** situation is attained with ***α*** equal to the new value of ***s***·***y*** with ***y*** < ***y*A**. That is, the equality between ***α***/***y*** and ***s*** is maintained. But this is again a merely transitory equilibrium so that this process continues. Thus, the economy implodes.

The only lower limit in Harrod’s model is with zero real GDP. It is possible, however, for this “centrifugal” movement to be blocked by exogenous changes, such as those due to Keynesian “animal spirits” (either raising ***ε*** and $\hat{y}$ or the expected **gz**), decreases in interest rates, expansionary fiscal policy, or an exogenous export boom. But the automatic movement limned above describes the “normal” or endogenous downward movement away from Harrod’s knife-edge moving equilibrium. Not surprisingly, Harrod’s story ignored the endogenous “floors” on the value of **g** of the sort later discussed by Hicks (1949) along with the role of automatic stabilization due to variations in the government budget and the like. In a more complete analysis, these should play a role in understanding the results of falling below the warranted growth path.

For the second case (point **B** with high capacity use), jump to step (b). If the economy starts with ***y*B**, expectations and investment behavior imply that both $\hat{y}$ and the expected value of **gz** rise. So ***α*** and ***y*** both rise, moving upward away from the warranted moving equilibrium at **W**. What’s different here from the previous case is that non-NC asymmetry plays a role: rising capacity use and the resulting increase in the prevalence of bottlenecks[[25]](#endnote-25) likely induces inflation, damping upward adjustment in real terms. Of course, as noted above, the inflation barrier at full capacity use is moderated to the extent that businesses hold unused capacity to allow flexibility. However, as discussed next, the inflation barrier at full employment of labor-power is qualitatively different. In the present model, unlike those of economies with “unlimited supplies of labor,” the supply of labor-power is inelastic.

* 1. Harrod and Solow’s Steady State.

As noted, Solow describes the “knife edge” as concerning the relationship between the **gf** and **gw**. As part of his analysis, he implicitly assumed that **g** equal **gw**, essentially ignoring Harrod’s knife edge entirely. But a sketch of the role of **gf** in Harrod is required before Solow’s problem can be addressed.

* + 1. A Trinity of Growth Rates.

To Harrod, **gf** is “the maximum rate of growth allowed by the increase in population, accumulation of capital, technological improvement, and the work/leisure preference schedule, supposing there is always full employment in some sense.” In the usual interpretation, this is stated as saying that accumulation and technological change raise the productivity of employed labor-power (***μ***) while the population’s change and changes in the work/leisure choice can raise the labor force (the number of people willing and able to work for pay). Given a unique and constant value of the NAIRU (***u*F**), the latter determines the available labor force (**N**). Both variables raise full-employment output **F** = ***μ***·**N**.

Let ***n*** = the rate of growth of **N** while ***m*** = the rate of growth of ***μ***. Both of these vary cyclically but that variation is erased by measuring them at the benchmark set by ***u*F** (with **Y** = **F**), in parallel to the government’s calculation of its cyclically-corrected budget. This allows us to treat ***m***, ***n***, and ***u*F** not only as independent of each other but as exogenously given during in the HMR. Thus, Harrod’s “natural” growth rate is:

|  |  |
| --- | --- |
| **gf** = ***n*** + ***m*** assumed to be positive. | (11) |

Among other things, treating these coefficients as parameters rules out endogenous adjustment of the labor-power supply due to wage changes (cf. Solow, 1956). Harrod’s analysis also elides the possible endogeneity of **gF**. Empirical analysis suggests that

1. sustained low demand growth can reduce ***m*** following Verdoorn’s empirical generalization (“law”), (Michl, 1985); and that
2. persistent low employment can raise ***u*F** following the more famous hysteresis hypothesis (Hargreaves Heap, 1980) or discourage in-migration and thus reduce ***n***.

Conversely, in moderate cases that avoid serious inflation, high demand growth and employment can raise **gf**. But Harrod’s long-run analysis, like the later NC ones, ignores these possibilities. However, their changes might be introduced, perhaps in a period longer than the Harrodian Longer Run.

Turning to the HLR itself (defined by **gF** being constant), ***c***, ***s***, and thus **gw** can change due to exogenous variations and endogenous effects. Harrod’s analysis of such changes is restricted to situations that start in a situation with (i) **g** = **gw** with ***y*** = 1 (the HMR moving equilibrium) and (ii) **g** = **gf** with **D** = **F** (FE). As to the first, the existence of the knife-edge makes the first assumption seem more than a bit implausible. But starting with warranted growth simplifies the analysis tremendously and will be kept. As seen below, however, assumptions (ii) are not necessary. Further, as discussed in §II.A.2II.A.1, issues of changing ***c*** are irrelevant when discussing deviations from warranted growth.

First, start with point **1** in figure 1. The actual and warranted growth rates start equal to what Harrod called the “proper” rate (**gF**), which clearly refers to a growth rate that should be preferred by the policy-makers or the citizenry. Now, consider a *fall* of the warranted rate of growth relative to the actual rate of growth. This occurs due to a decrease of ***s*** (perhaps due to Keynesian stimulus that lowers government saving) or changes in other factors that Harrod listed. These changes break the assumed HMR equilibrium because it means that **gw** falls below **g**. In terms of figure 2, an independent rise of ***α*** has this effect if any increase of ***s*** is smaller (or if ***s*** falls). Since **g** > **gw**, an upward surge in growth results.

Of course, as shown by the growth path represented by the dashed arrow from **1** to **2**, the rub is that the economy exceeds what Hicks (1949) and others dubbed the “ceiling” on the growth rate set by **gf** so that **Y** > **F**. The inflationary gap causes accelerating inflation (Lerner, 1951) and/or deepening shortages of goods and services (as during World War II).[[26]](#endnote-26) In the end, Harrod’s conclusion makes sense: inflation and/or rampant shortages induce recessionary tendencies which “drag down” the actual growth rate, keeping its “average value over a term of years down to the natural rate” of growth. That is, the model approaches a point such as **3**.

Dropping assumption (ii), the expansionary process could instead start with **Y** < **F** and ***e*** < 1 (a recessionary gap) as at point **4**. As before, assume **g** = **gw** = **gf** initially. Then, if **g** rises above **gw**, output rises toward line **F** with ***e*** and ***y*** rising. Then, once **F** is exceeded, the inflationary story limned above applies in an unaltered form. Even before that, unless **Z** > **F**, bottlenecks that occur when **Y** > **Z** encourage inflation when **g** > **gw**. As before, this result is moderated if firms deliberately hold unused capacity.

This story fits that where macropolicy is used to attain line **F**. That policy is not really needed to handle the case of **Y** > **F** except to speed the economy’s fall down to **F**.

As with others of the Keynesian school sketched in part I.A, Harrod saw no forces that would *automatically* push or pull a low value of **Y** upward to equal **F**. In fact, again starting with **g** = **gw** = **gf** (as at point **3**) and provisionally reinstating assumptions (i) and (ii), any shock that *raises* **gw** can push the economy into a serious tailspin toward point **5**, even if at the start we see **Y** = **F**. The recovery from the inflationary gap that Harrod sketched might cause this kind of recessionary disaster.

In this case, warranted growth rate rises above **g**, due to rising ***s*** relative to ***α***. This change causes **g** < **gw**. As discussed above, falling off the knife edge then causes a downward spiral and perhaps deflation and/or a true depression. The necessary dropping of assumption (ii) does not change anything. But as Harrod notes, this spiral might be counteracted by profit booms, perhaps led by speculative bubbles as with the U.S. bubble economy of 2002 to 2006. Exogenous counteracting forces such as “animal spirits” (***ε*** or expectations of **gz**) or Keynesian stimulus might also save the day.

This entire discussion implies a stark contrast between the NC and Keynesian schools. Like Solow (1956), the NC school emphasizes the warranted growth rate, which can be boosted by raising ***s***. But such changes – as with when austerity programs that raise government saving – lower the actual rate of growth, just as Keynesian analysis suggests. This result fits actual austerity cases, such as that of Greece after 2009. Similarly, if a program lowers ***s***, that lowers the warranted rate of growth but increases the actual rate of growth, which is a positive result until **Y** exceeds **F**.

* + 1. Solow’s contrast between Warranted and “Natural” growth.

As noted, Solow (1956) implicitly ignored the possible differences of **gw** from **g**, and thus Harrod’s knife edge. In fact, it seems that the actual growth rate is determined by the warranted rate while investment is determined by saving. Even if we allow this to be true, we face two problems.

1. How can the steady-state equilibrium with **g** = **gf** (with the supply of and demand for labor-power growing at the same rate and ***e***constant) be attained?
2. Since “natural” growth can occur with ***e*** < 1 and **Y** < **F** (as in figure 1), how can the actual ***e*** can be nudged upward to equal 1?

One way to answer these questions is to assume that Say’s discredited axiom applies. More reasonably, one might assume that Samuelson’s optimistic NC/Keynesian “synthesis” applies, with macropolicy being continuously successful at maintaining FE. From the perspective of 2015, that story seems quite odd. Of course, the NC school solves these problems by assuming that factor prices are determined by freely-adjusting markets and of course that ***c*** varies with those prices. These stories are addressed in the next part of the paper.

1. the Body of the Paper (TBA).

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Table 1: Summary of Symbols Used.

|  |
| --- |
| **absolute numbers (levels).** |
| **D** | the quantity of Real NNP demanded. |
| **Y** | the quantity of real NNP supplied (production). |
| **Z** | Capacity-or capital goods-constrained real NNP (at ***y*** = 1).  |
| **F** | Labor-power constrained real NNP (i.e., at full employment, with ***e*** = 1). |
| **K** | The stock of fixed capital goods (assumed homogeneous and move in step with the stock of desired inventories). |
| **Kd** | the desired capital stock. |
| **I** | Net investment in fixed capital goods = ∆**K**. |
| **ID** | desired flow of net investment. |
| **S** | the level of net saving = ***s***·**Y**.  |
| **N** | the available labor force = (1 – ***u*f**)·**Np** |
| **ratios** |
| ***a*** | The accumulation rate, investment as a percentage of full-capacity output (**I**/**Z**).  |
| ***a*Y** | Net investment as a percentage of actual output (**I**/**Y**). |
| ***c*** | The capital-output ratio (Harrod’s ***C***), capital coefficient, or ICOR. |
| ***μ*** | The productivity of labor-power (**F**/**N**), as measured at ***u*** = ***u*f**. |
| ***s*** | The saving rate (**S**/**Y**), net of deprecation. |
| ***y*** | The capacity utilization rate = **Y**/**Z** and equal to **D**/**Z** in the HMR. |
| $$\hat{y}$$ | The expected capacity utilization rate. |
| ***u*f** | The NAIRU = the so-called “natural” or full-employment rate of unemployment. |

|  |
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| **growth rates** |
| **g** | the actual rate of growth of demand and real NNP = ∆**Y**/**Y** in the HMR. |
| **gz** | the growth rate of full-capacity output = ∆**Z**/**Z**. |
| **gw** | Harrod’s warranted rate of growth = ***s***/***c*** in the HMR. |
| **gf** | Harrod’s “natural” (labor-constrained) rate of growth = ***m***+ **n**.  |
| ***m*** | The rate of growth of ***μ***, as measured at ***u*** = ***u*f**. |
| ***n*** | the rate of growth of the available labor force (**N**), as measured at ***u*** = ***u*f**. |
| **abbreviations.** |
| FE | Full employment (with ***e*** = 1 and ***u*** = ***u*f**). |
| NC | Neoclassical or Walrasian. |
| HMR | The Harrodian Medium Run, with ***c***, ***s***, and **gf** constant.  |
| HLR | the Harrodian Longer Run, with **gf** constant. |
| KC | the Keynesian Cross model. |
| KSR | the Keynesian Short Run, with **K** and **Z** constant.  |
| RE | so-called “rational” expectations. |
| RF | the representative firm.  |
| **miscellaneous.** |
| ***e***() | the adaptive expectations function. |
| ***ε*** | the random element in expectations adjustment. |
| ***θ*** | the partial adjustment coefficient. |
| $$\hat{}$$ | indicator of the expected value of a variable.  |

1. Here and below, FE production and employment are equated to the levels associated with the NAIRU (***u*F**, the Non-Accelerating Inflation Rate of Unemployment), which is assumed to be not only unique but known during the HMR. Because not all of Solow’s variants of his model are characterized by FE, what I call the “NC” literature completely ignores all of Solow’s realistic qualifications that allow for less-than full employment. [↑](#endnote-ref-1)
2. Strictly speaking, the usual story involves a single “imperfection” (most often, sticky money wages). If there are two “imperfections” (i.e., deviations from the Walrasian ideal), applying the Theory of the Second Best tells us that it is possible that the two might cancel each other out. In microeconomics, for example, a monopolist’s restriction of output can to some extent cancel the effects of negative externalities. [↑](#endnote-ref-2)
3. My interpretation of what Harrod meant by “proper” differs from that of Hoover (2008). [↑](#endnote-ref-3)
4. The NC “growth accounting” literature approaches empirical issues by *assuming* that the NC aggregate production function and much of the NC vision of perfection central to Solow’s model apply in the real world. [↑](#endnote-ref-4)
5. In Leijonhufvud’s (1973) terms, Keynesians consider cases not only inside but outside of the “corridor” around FE while the NC school limits itself to the cases inside it. [↑](#endnote-ref-5)
6. Of course, Keynes himself (1936: ch.14) rejected the application of the Classical or NC theory of interest-rate determination when we cannot assume that **D** and **Y** always equals **F**. Oddly, this “loanable funds” framework is seen nowadays in textbooks, often grafted onto Solow’s model. To Keynes, in the short run, with **F** > **Y**, interest rates were determined almost entirely by existing stocks of paper assets (especially money) and the demands for them rather than by the flow-supply of and flow-demand for assets as in loanable funds lore. [↑](#endnote-ref-6)
7. This vision fits Friedman’s (1968) definition of the “natural” unemployment rate as “the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is embedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the costs of gathering information about job vacancies, and labor availabilities, the costs of mobility, and so on.” [↑](#endnote-ref-7)
8. New Classicals and New Keynesians differ in that the former see long-run adjustment as occurring instantaneously (but with random error). Like Friedman (quoted above in note 7), the latter add “imperfections” to the Walrasian utopia, slowing adjustment (by causing frictions) and/or adding supply-side inefficiencies that limit the level of **D**, causing accelerating inflation even before all resources are efficiently utilized. [↑](#endnote-ref-8)
9. See, for example, Summers (2013). [↑](#endnote-ref-9)
10. For simplicity, I follow Harrod to assume that planned fixed investment includes desired inventory accumulation. The normal ratio between these two helps to determine ***c***. [↑](#endnote-ref-10)
11. The use of different “runs” aims to describe what Robinson (1970) calls “historical time” (in contrast to the merely “logical” and totally reversible time of NC comparative statics) without the unaesthetic use of time subscripts. [↑](#endnote-ref-11)
12. Saving to buy a house in the future (for example) almost never creates an order for delivery of a house at some future time. It thus does not cause the fixed investment needed to produce it. [↑](#endnote-ref-12)
13. The NC assumption that saving determines investment seems based on either Say’s axiom or the more fundamental assumption that **Y** = **F** in the “long run.” [↑](#endnote-ref-13)
14. This follows Kalecki’s (1943) distinction between sectors in which prices are “cost-determined” and those primary-product sectors in which prices are “demand-determined,” initially ignoring the latter. Note that, average fixed costs fall steadily as ***y*** rises. This part of KSR analysis holds the prices of inputs constant. [↑](#endnote-ref-14)
15. This means that I deliberately emulate the NC school’s dismissal or willful ignorance of the Cambridge Capital Controversy which debunked the aggregate production function on the field of theoretical battle (cf. Samuelson, 1966). Aggregation of “capital” only makes sense as a sum of money values of capital goods. [↑](#endnote-ref-15)
16. Here, the line is drawn as having ***e*** < 1 in light of the inflationary consequences of exceeding **F**, though it could (in theory) be drawn with ***e*** > 1. [↑](#endnote-ref-16)
17. To his credit, Solow (1956: 93) noted that “No credible theory of investment can be built on the assumption of perfect foresight and arbitrage over time.” [↑](#endnote-ref-17)
18. The idea is that any given time **t**, people do not know what is happening in the economy until after the fact (if then). So they make their decisions *now* based on expectations developed in past periods. [↑](#endnote-ref-18)
19. Also, a war, a natural disaster, or deliberate scrapping may imply low **K–1**. The lower limit on net investment (Hicks, 1949) will be ignored here. [↑](#endnote-ref-19)
20. Domar (1946: 143-45) considered this case. [↑](#endnote-ref-20)
21. In a corn-corn model, so-called “moral” depreciation due to obsolescence or changes in the demand for the output produced using specific types of capital goods cannot play a role. [↑](#endnote-ref-21)
22. However, this logic is usually hidden by the use of ratios. For example, the RF’s microeconomic value of a ratio such as the accumulation rate **I**/**Y** = ***α*y** also equals the ***α*y** at the macro level. [↑](#endnote-ref-22)
23. For example, low $\hat{Y}$ can cause low spending, causing low **Y**. The RE theory only allows one-way causation from **Y** to $\hat{Y}$. [↑](#endnote-ref-23)
24. The ***c*** seems to be important in Harrod’s *Essay* because he discusses the knife-edge in terms of the relationship between ***c*** (= **K**/**Z** = **I**/Δ**Z**) and its *ex-post* value (his ***C*p** = **I**/∆**Y**). This is mathematically equivalent to my presentation, since Δ**Y**/Δ**Z** is related to the changes in ***y***. [↑](#endnote-ref-24)
25. “Bottlenecks” occur in an economy with heterogeneous products and “factors of production” (and one not ruled by a RF). As one sector hits full capacity use or the full employment of specialized labor-power, that imposes higher costs on firms in other sectors. As **D** approaches and then exceeds **F**, the bottlenecks become general and eventually universal. [↑](#endnote-ref-25)
26. This result also occurs if we start with Harrod’s initial situation and then **gf** falls. [↑](#endnote-ref-26)